BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

SECOND SEMESTER 2017-2018

MIDSEM Exam (Closed Book)

Course No: PHY F342 **Course Title: Atomic and Molecular Physics** **Date: 05.03.2018**

Max. Time: 90 mins. Total Marks: 90

Q1 (i) The radial part of the Schrodinger equation for one electron atom can be expressed as; $u_{E,l}(\rho) = e^{-\frac{\rho}{2}} \rho^{l+1} g(\rho)$, where symbols have their usual meanings. The function $g(\rho)$ can be expressed in terms of confluent hypergeometric function ${}_1F_1(a, c, z) = \sum_{k=0}^{\infty} \frac{(a)_k}{(c)_k} \frac{z^k}{k!}$. It satisfies following differential equation:

$\left[\rho \frac{d^2}{d\rho^2} + (2l + 2 - \rho)\frac{d}{d\rho} + (\lambda - l - 1)\right] g(\rho) = 0$, where, $\lambda = \frac{Z e^2}{4\pi\epsilon_0}\left(\frac{-\mu}{2\rho}\right)^{\frac{1}{2}}$. Show that $g(\rho)$ will be a polynomial in ρ and this will lead to quantization of energy. Find out the energy. (14)

(ii) Find out $\langle\frac{1}{r^2}\rangle_{1,0,0}$ for the one electron atom. (6)

Q2 (i) The matrix element $\widetilde{M}_{ab}$ for the stimulated emission for the transition $b \to a$ is given as; $\widetilde{M}_{ab} = \left\langle\psi_a\middle|e^{-i\vec{k}.\vec{r}}\, \hat{\varepsilon}.\nabla\middle|\psi_b\right\rangle$. Find the relation $\widetilde{M}_{ab}$ with the matrix element for the absorption process $a \to b$. (8)

(ii) Starting from $M_{ba}^D = \langle\psi_b|\,\hat{\varepsilon}.\nabla|\psi_a\rangle$ in Dipole approximation for the linearly polarized light, find out the term $M_{ba}^D = -\frac{m\,\omega_{ba}}{\hbar}\,\hat{\epsilon}\,.\overrightarrow{r_{ba}}$. (12)

(iii) While solving $\overrightarrow{r_{ba}}$ for the spontaneous emission for the $2p \to 1s$ transition in the Dipole approximation, we encounter integral; $\int Y_{1m}^*(\theta,\phi)\cos\theta\; Y_{00}(\theta,\phi)\, d\Omega$. Solve this integral. (8)

Q3. (i) Find out the energy shift in ψ_{210} hydrogenic orbital due to the Hamiltonian $H = \frac{\pi\hbar^2}{2\,m^2c^2}\left(\frac{Z\,e^2}{4\pi\epsilon_0}\right)\delta(\vec{r})$. (6)

(ii) Find out the hyperfine levels of deuterium atom for n=2 level. The spin of the nucleus of the deuterium is 1. (8)

(iii) Draw the allowed dipole transitions in the hyperfine levels of deuterium for n=2 level. (8)

Q4 (i) Let $\mathbf{S_1}$ and $\mathbf{S_2}$ are the two spin operators of the two electrons and $\mathbf{S} = \mathbf{S_1} + \mathbf{S_2}$ is the total spin of the electrons. Similarly, $S_z = (S_z)_1 + (S_z)_2$. Solve following Eigen value problem; $S^2\chi$ and ; $S_z\chi$, where χ is singlet state of the two electrons atom. (8)

(ii) Explain why in the central field approximation the degeneracy of energy with respect to l is broken for the genuinely excited state of the two electrons atom. (5)

(iii) The ground state energy of some two electrons atom (ion) is found to be -21.88 atomic unit in the first order perturbation theory. Find out Z and the energy of the 2^3P excited state in the zero-order approximation. (7)

First few spherical harmonics: $Y_{0,0} = \frac{1}{(4\pi)^{\frac{1}{2}}}$, $Y_{1,0} = \left(\frac{3}{4\pi}\right)^{\frac{1}{2}}\cos\theta$, $Y_{1,\pm} = \mp\left(\frac{3}{8\pi}\right)^{\frac{1}{2}}\sin\theta\, e^{\pm i\phi}$

Useful relations: $S_x\,\alpha = \frac{\hbar}{2}\beta, S_x\,\beta = \frac{\hbar}{2}\alpha,\quad S_y\,\alpha = \frac{i\hbar}{2}\beta, S_y\,\beta = -i\frac{\hbar}{2}\alpha,\quad S_z\,\alpha = \frac{\hbar}{2}\alpha, S_z\,\beta = -\frac{\hbar}{2}\beta$

$R_{n,l} = -\left\{\left(\frac{2Z}{n\,a_\mu}\right)^3 \frac{(n-l-1)!}{2n\,[(n+l)!]^3}\right\}^{\frac{1}{2}} e^{-\rho/2}\rho^l L_{n+l}^{2l+1}(\rho)$, with $\rho = \frac{2Z}{n\,a_\mu}r, a_\mu = a_0\frac{m}{\mu}$

$L_0(\rho) = 1, L_1(\rho) = (1-\rho)$, and, $L_{q+1}(\rho) + (\rho - 1 - 2q)L_q(\rho) + q^2 L_{q-1}(\rho) = 0$.

$\int_0^\infty x^n \exp(-ax)dx = \frac{n!}{a^{n+1}}$, $\frac{1}{r_{12}} = \sum_{l=0}^{\infty}\frac{r_<^l}{r_>^{l+1}}P_l(\cos\theta)$, $\lim z \to \infty$, $F_1(a,c,z) \to \frac{\Gamma(c)}{\Gamma(a)} e^z z^{a-c}$