BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI.

SEMESTER II 2021 - 2022

PHY F342: Atomic and Molecular Physics: Mid-Semester Examination **(Closed Book)**

Date : 14 March 2022

Max. Marks : 90 **Duration : 90 Mins**

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1. The radial wavefunction $R_{n\ell}(r)$ is given as: $R_{n\ell}(r) = A_{n\ell}\ r^{\ell}\ \mathrm{e}^{-r/na_0} \sum_{k=0}^{n-\ell-1} b_k\ r^k$, where, $A_{n\ell}$ is normalization constant, n, ℓ are principal and orbital quantum number respectively, and a_0 is the Bohr radius. The expansion coefficients follow the recursion relation:
$$b_k = \Big[\frac{2\lambda(k+\ell) - 2/a_0}{k(k+2\ell+1)}\Big] b_{k-1}.$$
here, $\lambda = 1/na_0$. Obtain the normalized radial wavefunction for $2s$ orbital of Hydrogen atom, by first calculating required expansion coefficients b_k.
[Hint: $\int_0^\infty (1-qx)^2 e^{-2qx} x^2 dx = 1/(4q^3)$; $\int_0^\infty x^n e^{-qx} dx = n!/q^{n+1}$]
[20]

2. If the Hamiltonian of system with perturbation is given as $H = H_0 + \lambda H'$, where H_0 is unperturbed Hamiltonian and H' is the perturbation with λ being the control parameter. Under the non-degenerate perturbation theory calculate the first order energy correction to state k i.e. E_k^1. The $\{|\psi_k^0\rangle\}$ forms the complete orthonormal eigenstates of H_0.
[15]

3. Draw a schematic diagram of the fine-structure splitting for $n = 4$ state of Hydrogen atom. Calculate the correction in energy caused by the fine-structure splitting (in 10^{-5} eV) of the emitted photon during the transition $4D_{3/2} \to 2P_{1/2}$. Also comment on the helicity of the emitted photon. The fine-structure energy correction of the levels is given as,
$$\Delta E_{nj} = \frac{E_n \alpha^2}{n^2}\Big[\frac{n}{j+1/2} - \frac{3}{4}\Big],$$
where E_n is energy of the n^{th} level of H atom as obtained by solving Schrödinger equation, $\alpha \approx 1/137$ is fine structure constant.
[15]

4. In the normal Zeeman the hamiltonian is expressed as $H_{zeeman} = H_0 + H'$, where $H_0 = \mathbf{p}^2/2m - e^2/4\pi\varepsilon_0 r$ is the unperturbed hamiltonian. According to classical electrodynamics the interaction energy of the magnetic dipole (moment $\vec{\mu}$) in the presence of magnetic field is given by $H' = -\vec{\mu}\cdot\mathbf{B}$, construct the hamiltonian H_{zeeman}. The Schrödinger equation for the H atom in the presence of the constant magnetic field $\mathbf{B}$ is given as:
$$H_{zeeman}\ \psi(q) = E\psi(q),$$
where, $\psi(q)$ is the spin-orbital. Calculate the energy of the emitted photon in the transition $3d \to 2p$ without magnetic field, then estimate the energies associated with the linear and circularly polarized photon if applied magnetic field is 2 Tesla. Draw the appropriate energy levels with the possible transitions.
[20]

5. (a) Draw the schematic diagram of radial wavefunction $R_{n\ell}(r)$ for $2p$, $3d$, $5s$, $4p$, and $4s$ states.
   (b) If a wavefunction of a state is given as $\phi = 2(1/2a_0)^{3/2}\ (1 - r/2a_0)\ \mathrm{e}^{-r/2a_0}\ Y_{00}(\theta,\phi)$, calculate $\langle\phi|\delta(\mathbf{r})|\phi\rangle$.
   (c) The transition rate of spontaneous emission from the states $|3\rangle \to |0\rangle$, $|3\rangle \to |1\rangle$, and $|3\rangle \to |2\rangle$ are respectively 10^8 sec^{-1}, 0.5×10^8 sec^{-1} and 0.25×10^8 sec^{-1}. Calculate the lifetime of $|3\rangle$.
   (d) What causes the hyperfine splitting of the energy levels? Will the $5s$ state of the hydrogen atom undergo hyperfine splitting, if no then why and if yes then why. Mention some applications related to hyperfine splitting.
[4 × 5]

Physical Constants:

$c = 2.99792458\times10^8\,\mathrm{m\,s^{-1}}$; $\hbar = 1.05457182\times10^{-34}\,\mathrm{J\,s}$; $\alpha = 7.29735257\times10^{-3} \approx 1/137$

$m_e = 9.109383701500\times10^{-31}\,\mathrm{kg}$; $e = -1.602176634\times10^{-19}\,\mathrm{C}$; $m_p = 1.672621923690\times10^{-27}\,\mathrm{kg}$