PHYSICS DEPARTMENT; BITS-PILANI, PILANI

2ND SEMESTER 2021 - 2022

NUCLEAR & PARTICLE PHYSICS (PHY F343)
Comprehensive Examination (PART - A ; Closed Book; 1.5 hr)
Date & Time - 14/05/2022 ; 8:00 AM - 9:30 AM; Max. Marks - 40

Please attempt all parts of a question sequentially. You can take self-convincing justified assumptions, if required.

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1. Answer the following questions supplemented with justified statements (a couple of logical lines and/or mathematical steps should be sufficient for the justification).

   (a) Calculate the approximate mass density of nuclear matter in gm/cm^3. Take the nuclear radius parameter $r_0 \simeq 1.2$ fm. (*Fun Fact : Nuclear matter density is approximately equal to the average density of a neutron star !*)

   (b) If N_{o-o}, N_{e-e} and N_{o-e} are the (observed) number of stable nuclides (o/e denotes odd/even nucleon number), which among the following correctly represents their order (i) $N_{o-o} < N_{e-e} < N_{o-e}$ (ii) $N_{o-o} < N_{o-e} < N_{e-e}$ (iii) $N_{e-e} < N_{o-e} < N_{o-o}$ (iv) $N_{o-o} \simeq N_{e-e} \simeq N_{o-e}$.

   (c) Consider the Hamiltonian H of deuteron with the n-p potential of the form $V(r) = V_0(r) + f(r)S_{12}$. Where $V(r)$ and $f(r)$ are central in nature, and the operator S_{12} defines the non-central nature of the potential. Which among the operators L^2, S^2, J^2, J_z does not commute with H ?

   (d) A pair of β decaying nuclei X & Y have the $log(ft)$ values 3 and 12, respectively. Which among these nuclei has a more favorable β decay mode ?

   (e) Can the decay mode $\eta \longrightarrow 2\pi^0$ go through strong/EM interaction ?

   (f) Is the decay $\rho^0 \longrightarrow 2\gamma$ allowed ?

   (g) Assume you propose a theory auch that lepton number and baryon number are not necessarily conserved in a reaction. What would then be a possible decay mode of a free proton ? Note, you are not proposing the violation of other conserved quantities.

   [7 × 2]

2. Answer the following questions supplemented with a few mathematical steps [except (b)].

   (a) The energy (T_α) of α in the decay $Po^{214} \longrightarrow U^{210} + \alpha$ is 5.17 MeV. Relate T_α with the Q-value (Q_α) of the reaction and hence determine Q_α numerically. *If you need, take justified approximation, but at least don't assume* $Q_\alpha = T_\alpha$!

   (b) (Gell-Mann's eightfold way) Arrange all the ground state hadrons having spin-parity $J^p = 1^-$ and $3/2^+$ in a geometrical pattern by taking three lightest quark flavors (u,d,s and the anti-quarks) only. Note, the standard symbols should be used while naming the hadrons (*Nobel prize is not awarded for just renaming a particle !*).

   (c) Consider the reaction $p + p \longrightarrow p + p + p + \bar{p}$, where one of the initial protons is stationary (target) and the other proton (projectile) approaches the target exactly at *threshold energy.* Find $\sqrt{s}$ of the reaction in MeV. Recall the definition, $s = (p_1 + p_2)^2$.

   [3 × 5]

3. In a single particle shell model, assume the form of the potential to be a 3d harmonic oscillator $V_{ho} = \frac{m\omega^2 r^2}{2}$. The single particle energy level for such potential is given by $E_{nl} = \hbar\omega(\Lambda + 3/2)$. Where $\Lambda = 2n + l - 2$ is a non-negative integer with $n = 1, 2, 3, ...$ and $l = 0, 1, 2, ...$

   (a) For a fixed value of Λ, workout the necessary algebra to determine the degeneracy N_Λ of the an energy state (N_Λ is the maximum no of nucleons which can be accommodated in the state "Λ").

   [6]

   (b) Now take the potential $V = V_{ho} - f\ (\vec{l}\ .\ \vec{s})$. Where $f(> 0)$ is a constant and $\vec{l}$ & $\vec{s}$ are the single particle orbital and spin angular momentum respectively. Show pictorially the splitting of $\Lambda = 3$ energy state due to the *l-s* coupling. Use standard spectroscopic notations for the various levels.

   [5]

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1. The nucleus Si^{27}_{14} decays to Al^{27}_{13} by β^+ emission with a maximum kinetic energy of β^+ as 3.48 MeV. Using this data determine the coulomb energy coefficient a_c (in MeV). Assume neutrino mass $m_\nu = 0$ and ignore recoil energy of the daughter nucleus. [*Recall that the coefficient a_c appears in the expression of binding energy,* $B(A,Z) = -a_c\ \frac{Z(Z-1)}{A^{1/3}} + ...$]

   [10]

2. Take the n-p potential in the deuteron problem as a spherically symmetric well of depth V_0 and range $R_0 = 2$ fm. Assume that the binding energy of the deuteron is 9 MeV. Using the boundary condition and the continuity condition on the radial solution $u(r)$ (take $l = 0$), obtain a transcendental equation in the form $x\cot(x) = c$. Find the constant c and hence using the *input* given below, determine the depth of the potential V_0.
   Input : If you get the value of c correctly, then $x \simeq 2$ is supposed to be a solution of the transcendental equation.

   [6 + 4]

3. In a nuclear β decay, the $\beta^\pm$ spectrum can be written as $\lambda(T_\beta)dT_\beta \propto (T_\beta + m_e)(Q - T_\beta)^2\sqrt{T_\beta(T_\beta + 2m_e)}\ dT_\beta$. Here neutrino mass is assumed to be zero and also, neglecting the coulomb correction as well as the recoil energy of the daughter nucleus. For a cleaner look, c = 1 unit is used.

   (a) Now take $m_\nu \neq 0$ and obtain the expression $\lambda(T_\beta)dT_\beta$ in terms of Q, T_β and the rest masses of the particles. By the way, don't bother about the constant pre-factors.

   (b) For $m_\nu = 0$, what is value of the slope $\frac{d\lambda(T\beta)}{dT_\beta}$ at T_{max} ? Where T_{max} is the maximum kinetic energy of $\beta^\pm$.
   [Fun Fact: Non-zero m_ν changes the slope completely and hence 'predicts' the non-zero mass of the neutrino.]

   [8 + 2]

4. You are (perhaps) aware that Δ^0 belongs to a member of the baryon decuplet family. Answer the following questions related to this baryon.

   (a) Write down the flavor state $|\phi_f >$ and all the spin states $|\chi(J, M_J) >$ of this baryon (no need to derive). For convenience, use arrow (↑ and ↓) in a meaningful way.

   (b) Using the quark model, obtain the expression of the magnetic moment μ_{Δ^0} for the maximum spin projection state. The magnetic moment should be expressed in terms of the constituent quark magnetic moments μ_q.

   [(6 + 4)]

ALL THE BEST