

Please attempt all parts of a question sequentially. Use the data appended below, if required.

1. Answer the following questions supplemented with clean and precise logical/mathematical steps. **You will miss marks if you miss the important steps.**

- n-n bound state may exist in superfluid neutron systems as n-n Cooper pairs. Prove/disprove that the symmetrization of the n-n state allows (i) $L = 1, S = 1$ and (ii) $L = 2, S = 1$.
- Calculate the approximate mass density of nuclear matter in g/cm^3 in a nucleus.
- Using atomic mass data, calculate the Q-value (in MeV) for β^+ decay of Cu_{29}^{64} nuclide. Take $m_\nu = 0$.
- The wave function describing n-p scattering can be written as, $\psi(r, \theta) = \psi_{inc} + \psi_{sc} = e^{ikz} + f(\theta) \frac{e^{ikr}}{r}$. If $f(\theta) = A \cos \theta$, (A is a constant factor), obtain the total cross-section σ .
- For s-wave (i.e., $l = 0$) n-p scattering, the total cross-section σ can be expressed as, $\sigma = \frac{4\pi\hbar^2}{m_p} \frac{1}{E + |E_B|}$. Where E and $|E_B|$ are the scattering energy and the binding energy of the n-p system, respectively. At zero scattering energy limit, the observed cross-section is 21 barn. If you believe the observed cross section can be explained only from the triplet state scattering, what value of binding energy E_B do you expect for the n-p triplet state?
- Consider the alpha decay chain, $A \rightarrow B \rightarrow C$ with decay constants of A and B are λ_1 and λ_2 , respectively. At any time t , the number of A and B are given by $N_1(t) = N_1(0)e^{-\lambda_1 t}$ and $N_2(t) = \frac{\lambda_1 N_1(0)}{\lambda_2 - \lambda_1} [e^{-\lambda_1 t} - e^{-\lambda_2 t}]$. Obtain the expression of $t = t_m$ when N_2 becomes maximum. If the mean-life time of A and B are 4 hr and 8 hr, respectively, determine the numerical value of t_m .

[4 + 4 + 4 + 4 + 4 + 8]

2. Answer the following questions supplemented with the required mathematical steps.

- Assume the set of isobars with the mass number $A = 51$. Do the necessary algebra and determine the atomic number Z_0 of the most stable element (stable against β^\pm decay) among the isobars.
- The magnetic moment of deuteron ($J = 1; S = 1$) can be written as $\mu_d = (\mu_p + \mu_n) - (\mu_p + \mu_n - 1/2) \langle L_z \rangle$. Where, $L_z = \frac{\vec{L} \cdot \vec{J}}{J^2}$. If P_0 and P_2 are the respective probabilities for the deuteron being in $L = 0$ and $L = 2$, obtain the expression of P_0 and P_2 in terms of μ_d, μ_p and μ_n . Show the algebra clearly. No need to put the numerical values of the magnetic moments.
- In α -decay, the decay probability is given as $T = e^{-G}$, with the Gamow Factor $G = a \left(\frac{8m_\alpha E_\alpha}{\hbar^2} \right)^{1/2} \left[\frac{\pi}{2} - 2 \left(\frac{R}{a} \right)^{1/2} \right]$. Where R is the radius of the daughter nucleus, and a is the distance corresponding to the classical turning point. Now, do a suitable modification of various parameters that appears in the Gamow factor and calculate (numerically) the decay probability $T = e^{-G}$ for the (hypothetical) decay $\text{Fr}_{87}^{200} \rightarrow \text{Tl}_{81}^{188} + C_6^{12}$. The Q-value of the decay is 28 MeV.

[8 + 12 + 12]

Symbols/Formulae/Data :

Standard/lecture class symbols have been used.

Example : Total angular momentum (J), spin (S), isospin (I), Pauli matrices (σ), etc.

You can use : $1 \text{ u} \equiv 931.5 \text{ MeV}$; $\hbar c \simeq 200 \text{ MeV-fm}$, $e^2/(4\pi\epsilon_0\hbar c) = 1/137$; $m_n c^2 = 939.565 \text{ MeV}$, $m_p c^2 = 938.272 \text{ MeV}$, $m_e c^2 = 0.51 \text{ MeV}$; radius parameter $r_0 = 1.2 \text{ fm}$; $1 \text{ barn} = 10^{-28} \text{ m}^2$

Binding energy : $B = a_v A - a_s A^{2/3} - a_c \frac{z^2}{A^{1/3}} - a_a \frac{(A-2z)^2}{A} + \delta a_p A^{-3/4}$.

The values of various co-efficients (in MeV) : $a_v = 16$, $a_s = 17$, $a_c = 0.69$, $a_a = 25$, $a_p = 35$.

Atomic masses of few elements :

$M(64, 27) = 63.9358 \text{ u}$, $M(64, 28) = 63.9280 \text{ u}$, $M(64, 29) = 63.9298 \text{ u}$, $M(64, 30) = 63.9291 \text{ u}$