

Date: 12/10/2017

Weightage: 30 %

Time: 90 Mints.

Max. Marks: 60

Note: This question paper has two parts: Part A and Part B. It contains total eight questions including both the parts. Marks are indicated in front of questions. All the symbols not defined here have their usual meaning.

Part A

Q1: For a two body decay process, $a \rightarrow 1 + 2$, decay rate is given by $\Gamma_{fi} = \frac{(2\pi)^{-2}}{2E_a} \int |M_{fi}|^2 \delta(E_a - E_1 - E_2) \delta^3(\vec{p}_a - \vec{p}_1 - \vec{p}_2) d^3p_1/2E_1 d^3p_2/2E_2$. Here $|M_{fi}|$ is a Lorentz invariant matrix element. Use above expression to obtain a formula for the decay rate of particle a in the centre-of-mass frame of reference in terms of integral of $|M_{fi}|^2$ over solid angle and p^* , where p^* is the momentum of outgoing particles in the center-of-mass frame. [8]

Q2: Assume an electron is moving under the effect of an electromagnetic field described by electromagnetic four potential A^μ . Write Klein-Gordon equation for its motion. Find an expression for the lowest order scattering amplitude T_{fi} (covariant form i.e. $|T_{fi}|'$ in CN notation) in terms of four current J_μ and potential A^μ . [8]

Q3: Obtain covariant form of the Dirac equation for a particle of mass m and moving with a momentum \vec{p} from its corresponding non-covariant form. By using the properties of the Dirac gamma matrices, find Dirac equation for adjoint spinor, $\bar{\psi}$. [8]

Part B

Q4: Draw Feynman's diagram for the lowest order two body scattering process, $e^- \mu^- \rightarrow e^- \mu^-$ (do not forget to indicate +ve direction of time!). Write its invariant amplitude, ignoring the spin of the particles involved, using QED Feynman's rules without spin. [8]

Q5: Obtain the commutator of the Dirac Hamiltonian \hat{H}_D with spin angular momentum \hat{S} : $[\hat{\alpha} \cdot \hat{p} + \beta m, \hat{S}]$. [8]

Q6: Find an expression for the Dirac four current, J^μ . For a particle moving with a momentum p along z -axis, show that $\hat{S}_z u_1(E, 0, 0, \pm p) = +\frac{1}{2} u_1(E, 0, 0, \pm p)$ and $\hat{S}_z u_2(E, 0, 0, \pm p) = -\frac{1}{2} u_2(E, 0, 0, \pm p)$. Here $u_1(E, 0, 0, \pm p)$ and $u_2(E, 0, 0, \pm p)$ are the Dirac energy momentum dependent particle spinors and \hat{S}_z is the z -component of spin operator. [2+2+4]

Q7: Show that:

(i) $\not{p}\not{p} = p^2$

(ii) $\bar{v}(\gamma^\mu p_\mu + m) = 0$ [6]

for a particle/antiparticle of mass m and moving with three momentum \vec{p} .

Q8: Compute the following:

(i) $[\hat{\sigma} \cdot (\vec{p} + e\vec{A})][\hat{\sigma} \cdot (\vec{p} + e\vec{A})]$

(ii) $\bar{v}^{(s)} v^{(s)}$ [6]