Birla Institute of Technology and Science - Pilani, Pilani Campus

Semester I (Session 2022-23)

Comprehensive Examination (Closed Book)

Particle Physics (PHYF 413)

Date : 21/12/2022 Weightage : 15 %

Time: 90 Mints. Max. Marks: 15

Q1: Assume a two body scattering process; $a+b \rightarrow 1+2$ (a) Write an expression for number of accessible states for outgoing particles possessing momentum in the range $\vec{p}$ and $\vec{p}+\vec{dp}$ i.e., dN in terms of the Dirac-delta function and $d^3\vec{p_i}$ (b) Write a relation between Lorentz invariant amplitude M_{fi} and non-Lorentz invariant amplitude H_{fi} (c) Use integral form of the Fermi's Golden rule and results of Part(a) and Part(b) to write an expression for scattering cross-section in integral form. [3]

Q2: Write Dirac equation in covariant form. Solve it for a Particle solution and obtain all four solutions (i.e., u_1, u_2, u_3 and u_4) in terms of energy E and momentum components p_x, p_y and p_z. [3]

Q3: Define Pauli tensor $\sigma^{\mu\nu}$ in terms of Dirac gamma matrices. Write all 16 bilinear covariant quantities. Obtain transformation properties of the $\bar{\psi}\gamma^5\psi$ under Lorentz and parity transformations, where $\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3$. [3]

Q4: Draw all possible channel (tree level) Feynman's diagrams (indicating the direction of time and space) for Bhabha scattering process. Use full QED Feynman's rules to write an expression for the invariant amplitudes, $-iM$ for all the diagrams. [3]

Q5: (a) Show that $\frac{d^3\vec{p}}{E}$ is Lorentz invariant. Here $d^3\vec{p}$ is a volume element in 3D-momentum space and E is the energy of the particle. (b) Show that: Trace$(\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma) = 4g^{\mu\nu}g^{\lambda\sigma} - 4g^{\mu\lambda}g^{\nu\sigma} + 4g^{\mu\sigma}g^{\nu\lambda}$. [3]

**** Best Wishes ****

Birla Institute of Technology and Science - Pilani, Pilani Campus

Semester I (Session 2022-23)

Comprehensive Examination (Open Book)

Particle Physics (PHYF 413)

Date : 21/12/2022 Weightage : 20 %

Time: 90 Mints. Max. Marks: 20

Q1: Assume two body scattering process; $A + B \rightarrow C + D$. Draw possible Feynman's diagrams. Use full QED Feynman's rules to write its invariant amplitude $-iM$. [4]

Q2: Consider a Lorentz invariant amplitude; $M = [\bar{u_3}\Gamma_1 u_1][\bar{u_4}\Gamma_2 u_2]$ for a two body scattering process, where Γ_1 and Γ_2 have the property; $\Gamma^\dagger = \gamma^0\Gamma\gamma^0$. Assuming unpolarized scattering process and using Casimir's trick obtain the expression for $\Sigma_{allspins} < |M|^2 >$ in terms of the Trace of products of different Dirac gamma matrices. [4]

Q3: For a particular scattering process with unpolarized beam of particles with the same incoming and outgoing momentum; $\Sigma_{allspins} < |M|^2 >$ is given by: $\Sigma_{allspins} < |M|^2 >= e^4[\frac{t^2+s^2}{u^2}]$, where s, t and u are Mandelstam variables. (a) Use the appropriate expression for s, t and u to obtain the value of $\Sigma_{allspins} < |M|^2 >$ in terms of angle of scattering θ and momentum say k and energy E in the CM frame of reference. (b) How will you determine the diffeential and total scattering cross-section [no need to do the integral !]. [4]

Q4: Show that the following results related to the Dirac gamma matrices: (a) Trace of the product of three gamma matrices is zero. (b) $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$. (c) Use $\gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0$ to show that $\gamma^\mu = \gamma^0\gamma^{\mu\dagger}\gamma^0$. [4]

Q5: Consider antiparticle solutions of the Dirac equation, v_1 and v_2. (a) Use relativistic normalization condition to obtain the normalization constant, N. (b) Obtain v_1 and v_2 for an antiparticle moving along z-axis with momentum $|p|$. Use v_2 thus obtained to show that $S_z v_2 = -\frac{1}{2}v_2$, where $S_z = \frac{1}{2}\Sigma_z$. [4]

**** Best Wishes ****