Birla Institute of Technology and Science - Pilani, Pilani Campus

Semester I (Session 2023-24)

Comprehensive Examination (Closed Book)

Particle Physics (PHYF 413)

Date : 12/12/2023 Weightage : 15 %

Time: 90 Mints. Max. Marks: 15

Note: (i) Write your answers clean and precise.

(ii) All symbols used here have their usual meaning. In case of any issue, ask to Invigilator/Instructor.

Q1: Write Dirac equation for an electron moving under the action of e.m. field represented by four potential A^μ. Use $T_{fi} = -i\int J_\mu A^\mu d^4x$ to obatin an expression for the matrix element T_{fi} for a scattering process; $e^-\mu^- \to e^-\mu^-$. Here J_μ is the electron four current given by $J_\mu = -e\bar{\psi}_f\gamma_\mu\psi_i$. [3]

Q2: Solve Dirac equation for an anti-particle solution and obtain all four solutions (i.e., v_1, v_2, v_3 and v_4) in terms of energy E and momentum components p_x, p_y and p_z. [3]

Q3: Define Pauli tensor $\sigma^{\mu\nu}$ in terms of Dirac gamma matrices. Write all 16 bilinear covariant quantities. Obtain transformation properties of the $\bar{\psi}\psi$ and $\psi^\dagger\psi$ under Lorentz and parity transformations. [3]

Q4: Draw all possible channel (tree level) Feynman's diagrams (indicating the direction of time and space) for Moller scattering process ($e^-e^- \to e^-e^-$). Use QED Feynman's rules (with spin included) to write an expression for the invariant amplitudes, $-iM$ for all the diagrams. [3]

Q5: (a) Show that $\frac{d^3\vec{p}}{E}$ is Lorentz invariant. Here $d^3\vec{p}$ is a volume element in 3D-momentum space and E is the energy of the particle. (b) Obtain $[H_D, L]$, where H_D is the Dirac Hamiltonian, $H_D = \alpha.\mathbf{p} + \beta m$ and L is the orbital angular momentum of the Dirac particle. [3]

**** Best Wishes ****

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Q1: Assume two body scattering process; $A + \bar{A} \rightarrow A + \bar{A}$. Draw possible Feynman's diagrams. Use full QED Feynman's rules to write corresponding invariant amplitude $-iM$. Here $\bar{A}$ stands for corresponding antiparticle. [4]

Q2: Consider a Lorentz invariant amplitude; $M = [\bar{v_3}\Gamma_1 v_1][\bar{u_4}\Gamma_2 u_2]$ for a two body scattering process, where Γ_1 and Γ_2 have the property; $\Gamma^\dagger = \gamma^0\Gamma\gamma^0$. Assuming unpolarized scattering process and using Casimir's trick to obtain the expression for $\Sigma_{allspins} < |M|^2 >$ in terms of the Trace of products of different Dirac gamma matrices. [4]

Q3: Answer the following:

(a) Write Pauli-Weisskopf prescription for Dirac four current.

(b) Can you write a result (identity) which tells electron interacts among themselves not only via its charge but also via its spin angular momentum?

(c) Use definition of the Pauli's tensor $\sigma^{\mu\nu}$ and by taking appropriate values of μ and ν, relate it to the components of the spin angular momentum, S_x, S_y and S_z.

(d) Write an expression for propagator used to write an invariant amplitude for Compton scattering, pair annihilation and pair production processes. [4]

Q4: Show that the following results related to the Dirac gamma matrices: (a) $\mathrm{Tr}(\gamma^\mu\gamma^\nu\gamma^\lambda\gamma^\sigma\gamma^\delta) = 0$ (b) $\gamma^5\gamma^\mu = -\gamma^\mu\gamma^5$. (c) $\not{p}\not{p} = p^2$ [4]

Q5: Consider antiparticle solutions of the Dirac equation, v_1 and v_2. (a) Use relativistic normalization condition to obtain the normalization constant, N. (b) Obtain v_1 and v_2 for an antiparticle which is at rest. (c) Use v_1 thus obtained to show that $S_z v_1 = \frac{1}{2}v_1$, where $S_z = \frac{1}{2}\Sigma_z$. [4]

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