BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI
First Semester, 2022-2023
Comprehensive Examination - Part -A (Closed Book)
General Theory of Relativity and Cosmology (PHY F415)
Date: 27.12.2022 **Time:** 90 min **Maximum Marks.:** 60

Answer any four questions

1. Show that of all the circles of latitude on a sphere, only the equator is a geodesic. [15]

2. (a) Write down the geodesic equation in its standard form for the 3-D Euclidean space.[5]
(b) Show that if a general parameter $t = f(s)$ is used to parameterize a straight line in Euclidean space, then the geodesic equation takes the form.

$$\frac{d^2u^i}{dt^2} + \Gamma^i_{jk}\frac{du^j}{dt}\frac{du^k}{dt} = h(s)\frac{du^i}{dt},$$

where $h(s) = -\dfrac{d^2t}{ds^2}\left(\dfrac{dt}{ds}\right)^{-2}$.

Deduce that this reduces to the standard form if, and only if, $t = As + B$, where A, B are constants ($A \neq 0$). [10]

3. Consider an inertial reference frame K with coordinates (T, X, Y, Z). Considere K' whose coordinates are given as (t, x, y, z) and which is spinning with constant ω in counter-clockwise direction and whose origin and axis coincide with K at $t = 0 = T$ at which time their clocks are also synschronized. Using this information, establish the principle of equivalence. [15]

4. Write down the Einstein field equation. Recover the Poisson equation from the 00-component in the weak-field approximation. You may need to use the following Reiman tensor:

$$R^d_{abc} \equiv \partial_b\Gamma^d_{ac} - \partial_c\Gamma^d_{ab} + \Gamma^e_{ac}\Gamma^d_{eb} - \Gamma^e_{ab}\Gamma^d_{ec}.$$

5. (a)Define a 3-scalar, 3-vector, and a second rank 3-tensor. How are they different from 3-pseudo-scalar, 3-pseudo-vector, 3-pseudo-tensor? [7]
(b) Define Minkowski 4-scalar, 4-vector, second rank 4-tensor. Give an example of quantity that is (i) a Conserved but not a 4-scalar, (ii) 4-scalar but not conserved. [8]