BITS-Pilani K K Birla Goa Campus

Semester I 2022-23

Comprehensive examination

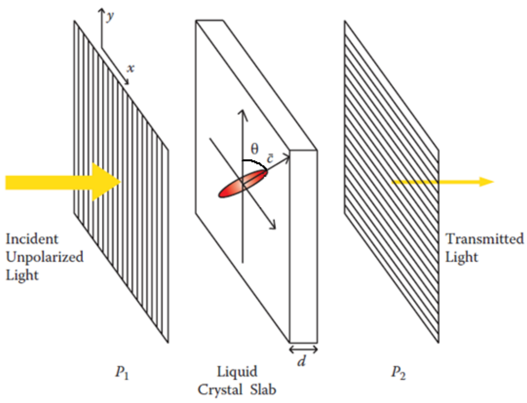
PHY F416 Soft Condensed Matter Elective course

Time: 180 mins Date: 29.12.2021 Full marks: 100 Type: Closed book/notes

---

(There are total 4 pages in the question paper. Please read the entire question carefully (don't miss any questions if it is on the next page!) before answering. Be careful about radius and diameter, and unit conversions. Some useful information is provided at the end of the question paper. You are allowed to bring a scientific calculator. Good luck!)

1. **Liquid crystals:**
   (a) Explain the working principle of a one-pixel liquid crystal display with a schematic diagram. (5)
   (b) How does a temperature sensor, based on liquid crystals work? (4)
   (c) What are 'homogeneous' and 'homeotropic' alignments for a liquid crystal? (2)
   (d) What are the different types of deformations exhibited by liquid crystals? Explain with schematic diagrams. (3)
   (e) What are topological defects in liquid crystals? (2)
   (f) In the following picture, unpolarized light is incident on the first polarizer P1 which is crossed with the second polarizer P2. In between the polarizers, is a liquid crystal slab of thickness d, birefringence Δn and its director vector $\vec{c}$ is oriented at an angle θ with respect to P1 as shown in the figure. Show that the intensity of the transmitted light through P2 is $I = I_0 sin^2 2\theta sin^2\left(\frac{\pi d}{\lambda}\Delta n\right)$ where I_0 is the intensity of the light incident on the liquid crystal slab. (6)



(Total =22)

2. **Polymers:**
   (a) Explain the concept of persistence length of a polymer. (2)

(b) Define what is a good solvent, bad solvent and theta solvent and draw schematic diagrams of a polymer configuration in each of these three solvents. (3)

(c) Derive the expression for the 'end-to-end distance' of an ideal polymer (a freely jointed chain) as a function of its degree of polymerization N. (3)

(d) A PEG molecule (Molecular weight: 20,000 Da) attached to planar, horizontal surface undergoes a transition from the coiled state to the brush state when the distance between individual molecules on the surface is approximately equal to the Flory radius. If we consider the PEG to be in a good solvent, calculate the surface coverage density for which this begins to happen. Take a = 0.35 nm and molar mass of each monomer unit = 44.44 Da (You can consider a square lattice type arrangement of the PEG molecules on the flat surface, where every vertex has a PEG molecule). (5)

(e) Define what is 'Reptation' and show that the terminal time $\tau_T \sim N^3$ where N is the degree of polymerization. (2+3=5)

(f) Describe an experimental technique that is widely used to characterize polymers. (3)

(Total = 21)

3. Gels and amphiphiles:

(a) The volume V of a linear hydrocarbon chain with n carbon atoms is given by V=(27.4+26.9n) × 10^{-3} nm^3 and its critical chain length is lc=(0.154+0.1265n) nm. An amphiphile has an anionic head group with an optimum head group area in aqueous solution of a_0 = 0.35 nm^2. Using the table given below, state what shape micelles are formed by amphiphiles with linear hydrocarbon tails with n = 10? (3)

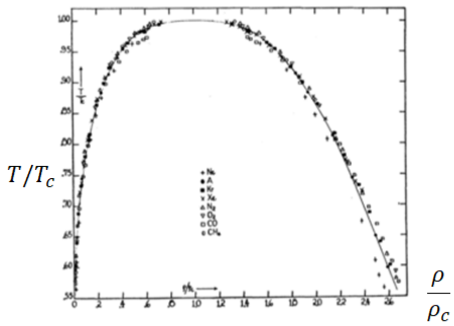
Phase	Packing Parameter P	Molecular Shape
Micellar	≤1/3	Cone
Cylindrical micelle (hexagonal)	≤0.5	Cone
Lamellar	0.5 to ~1	Cylinder
Inverted hexagonal	>1	Inverted cone
Inverted micelle	>1	Inverted cone

(b) How would the free energy of a surfactant molecule change as the head group area a_0 increases? (2)

(c) What are the differences between an emulsion and a microemulsion? Give two important practical applications of microemulsions. (2+2 = 4).

(d) Define the following with respect to a sol-gel phase transition. (i) Order parameter (ii) Critical exponent. (4)

(e) Guggenheim found the following graph for the liquid-gas phase transitions of several different chemical compounds. What was the significance of this experiment? Here ρ is the density and T_C is the critical point. (3)



(f) In a certain chemical cross-linking reaction involving a monomer that can react act $z = 3$ sites, the degree of reaction f obeys the second order rate law:

$$\frac{df}{dt} = k(1-f)^2$$

Where the rate constant $k = 4 \times 10^{-4} s^{-1}$. Calculate the time at which the gel point is reached as the system goes from the sol state to the gel state, considering at the gel point $f_c = \frac{1}{z-1}$. (4)

(Total = 20)

4. Colloids:

(a) Estimate the time taken by oxygen atoms of radius 152 pm (1 picometer=10^{-12} m) to diffuse in 3D in water (viscosity η = 8.9 x 10^{-4} Pa-s) at room temperature (25°C) a distance equal to:

i) the typical thickness of a bacteria, say 50 nm

ii) the typical thickness of a human being, say 24 cm.

iii) Verify from question that diffusive transport of oxygen from the environment to the lungs (via diffusion through skin across the thickness of the human body) is not an alternative to oxygen transport by red blood cells. (Assume the human body is mostly watery).

iv) What would be the time taken to diffuse through a distance of 24 cm in air (composed of pure oxygen gas)? Consider mass of oxygen atom = 2.6 × 10^{-26} Kg. (3+3+2+2=10)

(b) The electrophoretic mobility of colloid particles was measured in a 0.01 M NaCl salt solution and found to be μ = 8 × 10^{-8} $m^2s^{-1}V^{-1}$. Calculate (a) the Debye screening length λ_D and (b) the zeta potential ζ for particles of radius a = 200 nm. Take dielectric constant of water ϵ_r = 78 at 25°C and permittivity of free space ϵ_0 = 8.85 × 10^{-12} $m^{-3}kg^{-1}s^4A^2$. (5)

(c) Explain the differences between Rayleigh scattering and Mie scattering with one example for each. (2+2 =4)

(Total =19)

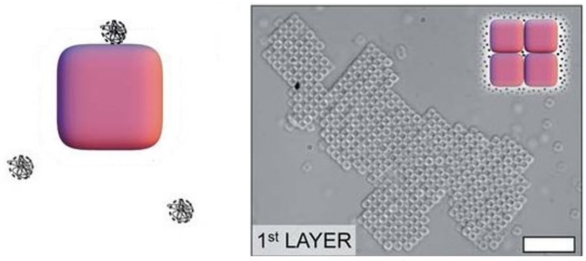
5. Others:

(a) What is Reynold's number? What are the differences between swimming at high Reynold's number (~10^4) and low Reynold's number (~10^{-4}) regimes? (2+2=4)

(b) Write four defining characteristics of soft materials. (4)

(c) What is the Casimir force? How does it differ from Van der Waals forces? (2+2=4)

(d) Look at the picture below:



The cubical SiO2 particles of size 1 μm form a crystal when polyethylene oxide (PEO) molecules of size 50 nm are added. Explain why this happens and describe the physics of the process. (3)

(e) For a linear viscoelastic material consider an oscillatory strain $\epsilon = \epsilon_0 sin\omega t$ applied on it, and let the resultant stress be σ. Plot a graph of $\boldsymbol{\sigma \ vs \ \epsilon}$ for i) a purely elastic ii) a purely viscous and iii) a viscoelastic solid with $\delta = 45°$. (3)
(Total = 18)

INFORMATION:

1. Boltzmann constant $k_B = 1.38 \times 10^{-23}$ m^2 kg s^{-2} K^{-1}
2. Avogadro number $N_A = 6.023 \times 10^{23}$.
3. 1 m = 10^6 µm = 10^9 nm = 10^{10} Å.
4. 1 g/cm^3 = 1000 Kg/m^3.
5. 1 M solution = 1 mole in 1 litre solution = 1000 moles in 1 m^3.
6. $U_{total} = F_{rep} + U_{int} = k_B T \frac{N^2 v}{2r^3} - - k_B T\ 2v\chi \frac{N^2}{2r^3} + constant = k_B T\ v \frac{N^2}{2r^3}(1 - 2\chi) + constant$
7. $\lambda_D = \left(\frac{\epsilon_r \epsilon_0 k_B T}{2e^2 N_A I}\right)^{1/2}$

   Here I is the ionic strength given as $I = \frac{1}{2}\sum_i z_i{}^2 c_i$

   z_i = Valence of ion type i
   c_i= concentration in moles/m^3
   N_A= Avogadro number = 6.023×10^{23}

8. $\mu = \frac{2\epsilon_r \epsilon_0 \zeta f(\lambda_D, a)}{3\eta}$

a is the radius of the particle, $f(\lambda_D, a)$ is Henry's function. If $\lambda_D \ll a$, then $f(\lambda_D, a) \sim 1.5$, giving Smoluchowski's formula: $\mu = \frac{\epsilon_r \epsilon_0 \zeta}{\eta}$. If $\lambda_D \geq a$, then $f(\lambda_D, a) \sim 1$, giving Huckel's formula: $\mu = \frac{2\epsilon_r \epsilon_0 \zeta}{3\eta}$.

9. Gravitational height of colloids = $\frac{k_B T}{\frac{4}{3}\pi a^3 g(\rho_2 - \rho_1)}$.
