Closed Book

BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

II SEMESTER 2021-22

Comprehensive Examination

Soft Condensed Matter Physics (PHY F416))

Date: 20-05-2022 **Max Time:** 120 min **Max Marks**: 80

IMPORTANT:

Mere deriving an expression is not a qualification of getting full marks. You have to explain the terms and equations that are appearing in the process of solving the problem.

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1. **(10 marks)** Consider a solid with a simple cubic structure, for which the interatomic potential $U(r)$ is given as,

$$U(r) = \frac{A}{r^n} - \frac{B}{r^m}$$

Having first derived expressions for the equilibrium separation a and the energy at equilibrium $-\epsilon$, derive an expression for the Young's modulus in terms of the bond energy ϵ and the equilibrium separation a. Assume that when a tensile stress is applied, the interatomic separations in directions perpendicular to the stress remain unchanged.

2. (**15 marks**) The phase behaviour of a certain liquid mixture can be described by the regular solution model. Consider a solution of two particles. By taking proper assumptions, prove that the free energy density of the mixing of solution is given by:

$$f(\phi) = \frac{k_B T}{v_c}[\phi \ln \phi + (1-\phi)\ln(1-\phi) + \chi\phi(1-\phi)]$$

where, $\phi = \frac{N_p}{N_T}$, $N_T = N_p + N_s$, and $v_c = \frac{V}{N_T}$ and χ is the interaction parameter.

3. (a) (**10 marks**) Show that the terminal velocity for sedimentation of colloidal spherical particles in a Newtonian fluid (water) is:

$$v = \frac{2}{9}\frac{R^2 \Delta\rho g}{\eta}$$

where η is the fluid's viscosity, $\Delta\rho$ is the difference in the fluid densities, and R is the radius of the particle.

(b) (**5 marks**) Find out the density of the polystyrene bead of radius is 2 μm. Given that the terminal velocity of the bead is 0.44 μm/s, the viscosity of water is 1.002×10^{-3} Pa-s, and the density of the bead is 1.05 g/cm^3. Take $\rho_w = 1$ g/cm^3.

4. **(15 marks)** Consider a polymer chain for which the probability distribution function of end-to-end distance is:

$$P(\boldsymbol{r}, N) = \left(\frac{2\pi N a^2}{3}\right) \exp\left(-\frac{3\boldsymbol{r}^2}{2Na^2}\right)$$

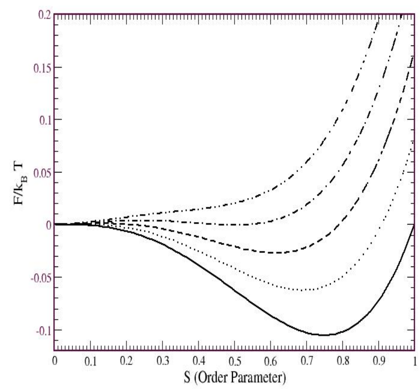
where, $\boldsymbol{r}$ is the end-to-end distance, N is the number of links each length a. We found that $\langle \boldsymbol{r} \rangle \sim N^{\nu}$ The ideal chain model suggests that $\nu = 0.5$. Find out the critical exponent ν, for real chain by considering the excluded volume effect.

5. **(5 marks)** An order parameter for a nematic phase can be described by the second order Legendre polynomial, $S = \frac{1}{2}\langle 3\cos^2\theta - 1\rangle$. Solve this equation for an order parameter $S = 1$ and $S = 0$. Comment on the obtained results.

6. **(5 marks)** The free energy per molecule transforming from isotropic to nematic phase according to Maier-Saupe theory is:

$$\Delta F = \frac{1}{2}uS^2 + k_B T \int f(\theta) \ln[4\pi f(\theta)] d\Omega$$

where, u is a parameter that expresses the strength of the favourable interaction between two neighbouring molecules, and S is the order parameter. $f(\theta)$ is the distribution function. With the help of the above equation, we plot the free energy as a function of the order parameter (as shown in the figure). The five different curves are for the different values of the coupling parameter, u. With proper justification, explain which curve is for the highest value of u.



7. Suppose that the unit vector $\boldsymbol{u}$ is isotropically distributed on the sphere $|\boldsymbol{u}| = 1$. Let $\langle ... \rangle_0$ be the average for this distribution

$$\langle ... \rangle_0 = \frac{1}{4\pi}\int d\boldsymbol{u}$$

Derive the following equation:

(a) (**5 marks**)

$$\langle u_z^2 \rangle_0 = \frac{1}{3}, \qquad \langle u_z^4 \rangle_0 = \frac{1}{5}$$

(b) (**5 marks**)

$$\langle u_z^{2m} \rangle_0 = \frac{1}{2m+1}$$

8. **(5 marks)** When we apply stress on the liquid crystal, it responds differently from the solids or liquid. Using a proper mathematical expression explain the kinds of distortions in liquid crystals under applied stress.

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Soft Condensed Matter Physics (PHY F416))

Date: 20-05-2022 **Max Time:** 60 min **Max Marks**: 40

IMPORTANT:

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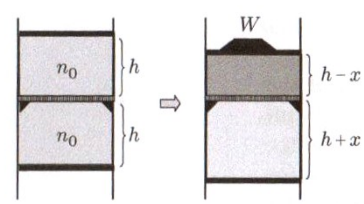
1. (**15 marks**) Find out the mean square displacement of the free Brownian particle using the following equations:

$$\langle v(t)v(t')\rangle = \frac{k_B T}{m}\exp\left(-\frac{t-t'}{\tau_v}\right)$$
$$\langle [x(t)-x(0)]^2\rangle = 2\int_0^t dt \int_0^{t'} dt_2 \langle v(t')v(0)\rangle$$

   Show that in the region $\mid t \mid >> \tau_v$, the mean square displacement is $2D \mid t \mid$.

2. A solution is held in a cylinder sealed by two pistons and divided into two chambers by a semi-permeable membrane. Initially, the concentrations of the two chambers are the same at number density, n_0, and the chambers have the same volume hA, where h is the height, and A is the cross-section of the chamber. A weight W is placed on top of the cylinder, causing the piston to move down.



   (a) (**15 marks**) Ignoring the density of the solution and assuming that the solution is ideal, find out the displacement x of the piston at equilibrium.

   (b) (**5 marks**) If we consider the density of the solution ρ, find out the displacement x of the piston.

3. **(5 marks)** Consider a colloid of charged spheres all of the radius 0.1 μm in an aqueous sodium chloride solution. Find out the Debye screening length for salt concentration of 10^{-5}, & 10^{-2} mol/dm^3. Compare your results for these two concentrations.