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BIRLA INSTITUTE OF TECHNOLOGY & SCIENCE, PILANI

II SEMESTER 2021-22

Mid-Semester Examination

Soft Condensed Matter Physics (PHY F416))

Date: 15-03-2022 **Max Time:** 90 min **Max Marks**: 90

IMPORTANT:
Mere deriving an expression is not a qualification of getting full marks. You have to explain the terms and equations that are appearing in the process of solving the problem.

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1. **(15 marks)** Consider a polymer that is pulled by an applied force, $\boldsymbol{f}$. The potential energy of the system is

$$E_f(\{\boldsymbol{r}_i\}) = -\boldsymbol{f}.\sum_i \boldsymbol{r}_i.$$

By defining the probability of finding the chain in a configuration $\{\boldsymbol{r}_i\} = (r_1, r_2...r_i)$, find out the average of $\boldsymbol{r}_i$ and show that it is,

$$\langle \boldsymbol{r}_i \rangle = r\frac{\boldsymbol{f}}{\mid \boldsymbol{f} \mid}\left[\coth(\xi) - \frac{1}{\xi}\right]$$

where $\xi = \frac{|\boldsymbol{f}|r}{k_B T}$.

2. Consider a homogeneous solution made of two components, *solute* and *solvent*.
   (a) (**7 marks**) Find out the Helmholtz free energy per unit volume $f(\phi)$.
   (b) (**7 marks**) Using the phase diagram, explain the condition in which a solution can be considered as a homogeneous/heterogeneous.
   (c) (**5 marks**) Define the spinodal and binodal lines on the phase diagram.
   (d) (**10 marks**) Using lattice model, derive an expression for $f(\phi)$.
   (e) (**6 marks**) Through the proper phase diagram, discuss the limit of solubility of one immiscible liquid in another in terms of the interaction parameter, χ.

3. **(10 marks)** Consider an $(m+1)$-component solution. Let M_j be the number of molecules of component j $(j = 0, 1, 2..)$ in the solution, and v_i be the specific volume of each component. The total volume of the solution is given by,

$$V = \sum_{j=0}^{m} v_j N_j$$

Assuming that v_j are all constant, show that the Gibbs free energy of the solution is given by:

$$G(M_0, ..., M_m, T, P) = PV + Vf(\phi_1, \phi_2, ..., \phi_m, T)$$

where $\phi_j = \frac{v_j M_j}{V}$ is the volume fraction of the j^{th} component.

4. (a) (**10 marks**) Consider a polymer of $N + 1$ monomers on a random walk. Find out the expression for the end-to-end distance of the polymer chain. Assume a constant bond length b between each monomer.
   (b) (**5 marks**) Discuss the shortcoming of the model and calculate the more accurate expression for the end-to-end distance using the concept of a statistical segment.

5. For polystyrene chains with a degree of polymerization 3.0×10^6 and average bond length 0.35 nm, find out the following:
   (a) (**5 marks**) the RMS value of end-to-end distance in a melt.
   (b) (**5 marks**) the RMS value of end-to-end distance in a dilute, good solvent, with a value of the interaction parameter, $\chi = 0$.

6. **(5 marks)** Consider a uniaxial elongation in a viscoelastic material at a point $P(x, y, z)$. Due to applied stress the points x, y, z are displaced to $x' = \delta^{-2}x,\ y' = \delta^{-2}y,\ z' = \delta^3 z$. The strain is defined as $\gamma(t) = \frac{1}{\delta(t)}$. Find out the velocity at point $P'(x', y', z')$.