Birla Institute of Technology and Science, Pilani
Mid-Semester Examination

PHY F420 (Quantum Optics) **Date: 17.03.2023** **Time: 90 Mins.**

Q1a) Consider an electromagnetic field in free space (free of charge and current). Show that it is possible to choose a gauge so that

$$\vec{\nabla}\cdot\vec{A} = 0 \;\&\; \phi = 0 \qquad (18)$$

b) Consider a vector field that satisfies the wave equation

$$\nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial\vec{A}}{\partial t} = \vec{0}$$

The Fourier expansion of the vector field with periodic boundary conditions is the following superposition of plane waves.

$$\vec{A}(\vec{r},t) = \sum_{\vec{k}}[\vec{A}_{\vec{k}}e^{i(\vec{k}\cdot\vec{r}-\omega t)} + \vec{A}^*_{\vec{k}}e^{-i(\vec{k}\cdot\vec{r}-\omega t)}]$$

Now suppose, the vector field is irrotational, i.e., $\vec{\nabla}\times\vec{A} = \vec{0}$.

Show that the plane waves will be longitudinal waves. Write down the above expansion by separating out the polarization vector from the coefficient vectors, $\vec{A}_{\vec{k}} \;\&\, \vec{A}_k$. (12)

Q2) The scattering of a photon from state ($\vec{k}\alpha$) to state ($\vec{k}'\alpha'$) by an atom is the following process.

$$|A>\otimes|n_{\vec{k}\alpha}, n_{\vec{k}'\alpha'}> \;\rightarrow\; |B>\otimes|n_{\vec{k}\alpha}-1, n_{\vec{k}'\alpha'}+1>$$

The interaction Hamiltonian between the atom and the field is

$$H^1 = -\hat{\vec{d}}\cdot\hat{\vec{E}}$$

a) By how much will the energy and momentum of the initial and final atomic states differ?

b) Argue (without any detailed calculations) that the above scattering process cannot occur in the first order of perturbation.

c) Argue (without any detailed calculations) that the scattering can happen in the second order of perturbation. Show that the transition probability will be proportional to the product of the photon numbers

$$n_{\vec{k}\alpha}(n_{\vec{k}'\alpha'}+1) \qquad (6 + 8 + 12)$$

Q3) Consider the time evolution operator in the interaction picture, $U_I(t)$. By using the differential equation that it satisfies, prove that U^+U is a constant (do not assume unitarity yet):

$$\frac{d}{dt}(U^{*}U) = 0$$

Hence, using the initial condition on U_I, prove that U_I is indeed unitary. (12)

Q4a) Consider the semi-classical Rabi oscillation between the states $|100>$ & $|210>$ of a hydrogen atom by an external field $\vec{E} = E_0\hat{z}\cos\omega t$. The matrix element $<100|z|210>$ is equal to $\sqrt{2}\frac{2^7}{3^5}a_0$, where $a_0 = 0.5A^0$ is the Bohr radius. The field strength E_0 is such that it corresponds to an intensity of $1w/m^2$. Calculate the Rabi frequency at resonance. (12)

b) Construct the Pauli operators $\hat{\sigma}_+$ & $\hat{\sigma}_-$ for a two level atom, with the two states denoted as $|g>$ & $|e>$ respectively. Calculate their actions on the two atomic states, $|g>$ & $|e>$. (10)