Birla Institute of Technology and Science, Pilani

Comprehensive Examination

PHY F420 (Quantum Optics) **Date: 18.05.2023** **Time: 3 Hours**

Closed-Book

Q1) For a single mod e-m field, calculate the vacuum fluctuation of the electric field, where the electric field operator is given by

$$\vec{E}_{\vec{k}\alpha} = i\sqrt{\frac{\hbar\omega_{\vec{k}}}{2\varepsilon_0 V}}\left(\vec{u}_{\vec{k}\alpha} a_{\vec{k}\alpha} - \vec{u}^{*}_{\vec{k}\alpha} a^{+}_{\vec{k}\alpha}\right) \qquad (12)$$

Q2) From the ratio of the emission to the absorption rate by an atom in the presence of photon of mode $(\vec{k}\alpha)$:

$$\frac{W_{em}}{W_{abs}} = \frac{n_{\vec{k}\alpha}+1}{n_{\vec{k}\alpha}}$$

derive the Planck distribution law: $n_{\vec{k}\alpha} = \dfrac{1}{e^{\hbar\omega/kT}-1}$ (12)

Q3) The fundamental definition of the parity operator is

$$\hat{\pi}|\,\vec{x}> = |-\vec{x}>$$

Derive the results:

$$\hat{\pi}\,\hat{\vec{x}}\,\hat{\pi} = -\hat{\vec{x}} \;;\; \hat{\pi}\,\hat{\vec{p}}\,\hat{\pi} = -\,\hat{\vec{p}} \qquad (12)$$

Q4) The interaction Hamiltonian for a two-level atom (Jaynes-Cummins model) is given as

$$\hat{H}_{int} = \frac{\lambda\hbar}{2}\left(\sigma_{+}a + \sigma_{-}a^{+}\right) \qquad (\sigma_{+} = 2|\,e><g\,|)$$

Assume that the time-evolving state of the atom-light system is the superposition:

$$|\,\psi(t)> = c_1(t)\,|\,e>\otimes|\,n> + \,c_2(t)\,|\,g>\otimes|\,n+1>$$

The time evolution equation in the interaction picture is

$$i\hbar\frac{d}{dt}|\,\psi(t)> = \hat{H}_{int}\,|\,\psi(t)>$$

obtain the coupled differential equations for the coefficients $c_1(t)\;\&\;c_2(t)$ (12)

Q5) Prove that the Glauber-Sudarshan P-function is normalized:

$$\int d^2\alpha \, P(\alpha) = 1 \tag{7}$$

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Open-Book

Q1a) Consider a photon, propagating along the z-axis, is in the polarization state

$$|\psi> = \frac{1}{2}|R> + \frac{i\sqrt{3}}{2}|L>$$

where $|R> \& |L>$ represent right and left circularly polarized photon states. The photon is passed through a polariser that passes linearly polarized photons, polarized along the x-axis and represented by the ket $|x>$. What is the probability that the photon will pass through the polariser? (8)

b) A completely unpolarised photon is a mixed quantum state, with equal mixture of the two linearly polarization states $|x> \& |y>$ (the photon is propagating along the z-axis). Write down the mathematical representation of such a state. Show that this mixed state is also an equal mixture of the two circularly polarization states $|R> \& |L>$. (10)

Q2a) Show that the Weyl transform of a Hermitian operator is a real function in the phase space.

b) Prove that if $\hat{A} \& \hat{B}$ are Hermitian operators, then $WT(\hat{A}\hat{B}) = (WT(\hat{B}\hat{A}))^*$.

c) Prove that the Weyl transform of an operator that is a function of position operator alone, is the same function of the position variable.

d) The Wigner distribution in quantum optics is defined as

$$W(\alpha) = \frac{1}{\pi^2}\int d^2\beta e^{\beta^*\alpha - \beta\alpha^*} Tr(\hat{\rho}\hat{D}(\beta))$$

Prove that the distribution is normalized:

$$\int d^2\alpha W(\alpha) = 1$$ (6 + 6 + 6 + 6)

Q3a) Prove that $D^+(\alpha)\hat{a}D(\alpha) = \hat{a} + \alpha \, ; D^+(\alpha)\,\hat{a}^+ D(\alpha) = \hat{a}^+ + \alpha^*$ (Use Baker-Hausdorff-Campbell lemma). (8)

b) Using the result of part (a), prove that the mean photon number in a squeezed displaced state $|\alpha,\eta> = D(\alpha)S(\eta)|0>$ is given by

$$<N> = <a^+a> = |\alpha|^2 + \sinh^2 r$$

where $r = |\eta|$ (10)

c) Calculate the fluctuation (uncertainty) in photon number. (10)

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