BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

FIRST SEMESTER: 2023-2024

<u>Midsem Test (Closed Book)</u>

Course No: PHY F421 **Course Title: Advanced Quantum Mechanics**

Date: 10.10.2023 Max. Time: 90 Mins. Total Marks: 30

Q1 (a) The radial equation for the scattering of a spinless particle of mass m from the reduced potential, $U(r) = \frac{2m}{\hbar} V(r)$, is given as $\left[\frac{d^2}{dr^2} - \frac{l\,(l+1)}{r^2} - U(r) + k^2\right] u_l(k,r) = 0$ (symbols have their usual meanings). Find out the solution $u_l\,(k,r)$ for $r \to 0$. (5)

(b) Starting from the above radial equation, find out the expression for the phase shift in the integral form in terms of the radial functions and the potential *U(r)*. (4)

(c) The Lippmann-Schwinger equation for the scattering theory is given as:

$\psi^{(+)}_{\boldsymbol{k_i}}\,(\boldsymbol{r}) = \exp(i\boldsymbol{k_i}\,.\,\boldsymbol{r}) - \frac{1}{4\pi}\int \frac{\exp(ik\,|\boldsymbol{r}-\boldsymbol{r}'|)}{|\boldsymbol{r}-\boldsymbol{r}'|}\, U(\boldsymbol{r}')\psi^{(+)}_{\boldsymbol{k_i}}\,(\boldsymbol{r})\, d\boldsymbol{r}'$, here symbols have their usual meanings.

(i) Find out the asymptotic behavior of $\psi^{(+)}_{\boldsymbol{k_i}}\,(\boldsymbol{r})$, given that the potential decays much faster than 1/r at the asymptotic distance. (5)

(ii) From the asymptotic behavior, find out the scattering amplitude. (2)

Q2 Consider a problem of coupled oscillations, discussed in MeOW, in which 2 identical masses *m* are loaded on a massless string at equidistance *l* with their ends are fixed (x=0 and 3*l*). The tension in the string is *T*. The eigen value problem is given as: $\ddot{x}_i + \omega_i^2\, x_i = 0$, *i= I, II* and $\omega_1 = \sqrt{\frac{T}{m\,l}}$ and $\omega_2 = \sqrt{\frac{3T}{m\,l}}$ and eigen basis are, $\left|I\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$ and $\left|II\right\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$. The initial values of the two masses are: $x_1\,(0) = 3\ cm\,, x_2\,(0) = 5\ cm.$

(a) Find out the position $x_1(t)$ and $x_2(t)$ from $|x(t)\rangle$, expanded in the basis $|I\rangle$ and $|II\rangle$.(4)

(b) Find out the propagator *U.* (2)

(c) Find out the matrix elements U_{21} & U_{22} in the old basis vectors ($|1\rangle = \begin{bmatrix}1\\0\end{bmatrix}, |2\rangle = \begin{bmatrix}0\\1\end{bmatrix}$). (3)

Q3 : (a) Find out the orthonormal basis from the basis:

$|I\rangle = \begin{bmatrix}2\\0\\0\end{bmatrix}$, $|II\rangle = \begin{bmatrix}0\\2\\1\end{bmatrix}$ and $|III\rangle = \begin{bmatrix}0\\5\\2\end{bmatrix}$ (3)

(b) Find, $\int_{-\infty}^{\infty} \frac{d}{dx}\delta(x)\sin x\, dx'$ (2)