BIRLA INSTITUTE OF TECHNOLOGY AND SCIENCE, PILANI

FIRST SEMESTER: 2023-2024

Comprehensive Exam: Part-A (Closed Book)

Course No: PHY F421 **Course Title: Advanced Quantum Mechanics**
Date: 09.12.2023 **Suggested** Time: 120 Mins. Total Marks: 25

Note: It contains two parts: Part 1 (Quiz/small answer like) and Part 2 (Descriptive type). Both the parts have to be answered in the same Answer Sheet (Please write Sheet-A on the top of the answer sheet). Once you submit the answer sheet A, you can start writing the Part-B, which is Open book in nature.

Part-1

Q1. The parity operation converts a right handed coordinate system to left handed. Write down the matrix element of the Parity operator in the space basis (Cartesian coordinate). (0.5)

Q2. Consider the matrix $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$,
(a) What is the nature of the matrix.
(b) Find out its eigenvalues. (0.5+1.0)
Q3. Express $\delta(x' - x)$ in the integral form. (0.5)

Q4. What is Green's function $G_0^{(-)}\,(k.\boldsymbol{r}.\boldsymbol{r}')$ (0.5)

Q5. An electron beam of energy 4 atomic unit (a.u.) is elastically scattered from an atomic target. The momentum transfer is 2 atomic unit. Find out (a) scattering angle and (b) total cross section for *l=0* case. (0.5+0.5)

Q6. Express the state vector and the operator (let us assume *A*) of some physical system in the Interaction picture with those in the Schrodinger picture. (0.5)

Q7. Express the transition probability (from the state $|i\rangle$ to $|n\rangle$) in the Interaction picture for any system in terms of that for the Schrodinger Picture. (0.5)

Q8. What is $c_1^n(t)$ for the constant perturbation (V(t)= V (constant) for $t \geq 0$ & $0, t < 0$). (1)

Q9. Consider three identical particles in the state $|k'\rangle,\ |k''\rangle, |k'''\rangle$. Form the symmetric and anti-symmetric states for the case when any two of the states are same (say $k'\rangle$ & $|k'''\rangle$) . (1)

Part-2

Q1. In the function space we can describe the action of this operator as $D|f\rangle = |df/dx\rangle$ is the ket corresponding to the function *df/dx.*

(a) What are the matrix elements of D in the $|x\rangle$ basis?
(b) Find the Hermitian Operator K from D operator and test its Hermitian properties.
(c) Write down the K in the x-basis.
(d) Find $\langle k|k'\rangle$, where $|k\rangle$ is plane wave. (1+2+1+2)

Q2. Starting from $f^{B1} = -\frac{1}{4\pi}\left\langle \phi_{k_f} \middle| U \middle| \phi_{k_i} \right\rangle$, for the Yukawa Potential $U(r) = U_0\,\frac{e^{-\alpha r}}{r}$, find

(a) Solve it to get f^{B1}, assuming the incident and scattered wave functions as plane waves.
(b) Find the differential cross-section.
(c) From part (b), find the total cross section. (3+1+1)

Q3. Consider a spin ½ system-say a bound electron, subjected to a *t*-independent uniform, magnetic field in the *z*-direction and, in addition, a *t*-dependent magnetic field rotating in the xy plane;
$\boldsymbol{B} = B_0\,\hat{z} + B_1\,(\hat{x}\cos\omega t + \hat{y}\sin\omega t)$, with B_0 & B_1 constant. Using $H = -\boldsymbol{\mu}.\boldsymbol{B}$, where $\boldsymbol{\mu} = \frac{\boldsymbol{e}}{\boldsymbol{m_e c}}\,\boldsymbol{S}$ (symbols have their usual meanings). ($S_x = \frac{\hbar}{2}\,\{(|+\rangle\langle-|) + (|-\rangle\langle+|)\}, S_y = \frac{i\hbar}{2}\,\{-(|+\rangle\langle-|) + (|-\rangle\langle+|)\}, S_z = \frac{\hbar}{2}\,\{(|+\rangle\langle+|) - (|-\rangle\langle-|)\}$)

(a) Split H in terms of the spatial and temporal dependence part.
(b) Find the frequency ω_{21}. (5+2)

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FIRST SEMESTER: 2023-2024

Comprehensive Exam: Part-B (Open Book)

Course No: PHY F421 **Course Title: Advanced Quantum Mechanics**

Date: 09.12.2023 **Suggested** Time: 60 Mins. Total Marks: 15

Q1. Let $\mathbf{S_1}$ and $\mathbf{S_2}$ are the two spin operators of the two electrons and $\mathbf{S}= \mathbf{S_1}+\mathbf{S_2}$ is the total spin of the electrons. Similarly, $S_z= (S_z)_1 + (S_z)_2$. Solve following Eigen value problem; $S^2\, \chi_2$ *and* ; $S_z\, \chi_2$, where $\chi_1 = \frac{1}{\sqrt{2}}\,[\,\alpha(1)\,\beta(2) - \alpha(2)\,\beta(1)]$ and the symbols have their usual meanings. (4)

Q2. (a) Obtain in first Born approximation the scattering amplitude, the differential and the total cross sections for scattering by the potential $U(r) = B\,\delta(r)$. (3)

(b) For a certain scattering event of electron-atom collision for 81.6 eV electron impact energy, the scattering amplitude is expressed as: $f(k,\theta) = 0.2\sin\theta + i\,(0.2\,k\cos\theta)$. Find out the total cross section from it (use atomic unit ; $e = m = \hbar = a_0 = 1, c = 137, 27.2\,eV = 1$). (2)

Q3. Consider a photo double ionization process in which two electrons are ejected from an atom following the absorption of a photon. Assuming that these electrons are ejected with the momenta $\hbar\boldsymbol{k_1}$ and $\hbar\boldsymbol{k_2}$ respectively. Find out the number of states for the energy intervals E_1+dE_1 and E_2+dE_2 with their directions into $d\Omega_1$ and $d\Omega_2$ of the momenta $\hbar\boldsymbol{k_1}$ and $\hbar\boldsymbol{k_2}$ respectively. (4)

(b) Find the energy flux for a monochromatic field with vector potential $A = 2\,A_0\,\hat{x}\cos\left(\frac{\omega}{c}\hat{z}.\boldsymbol{x} - \omega t\right)$ using classical electromagnetic field theory (Don not copy the results, derive it). Here symbols have their usual meanings. (2)

Useful relations:

$$S_x\,\alpha = \frac{\hbar}{2}\,\beta, S_x\,\beta = \frac{\hbar}{2}\,\alpha, \quad S_y\,\alpha = \frac{i\hbar}{2}\,\beta, S_y\,\beta = -\,i\frac{\hbar}{2}\,\alpha,\ \ S_z\,\alpha = \frac{\hbar}{2}\,\alpha, S_z\,\beta = \ -\frac{\hbar}{2}\,\beta$$