Birla Institute of Technology & Science – Pilani, K K Birla Goa Campus

Second Semester 2022-2023

Special Topics Statistical Mechanics Comprehensive Exam

PHY F423 May 10, 2023

Total Marks: 60 Duration: 180 Minutes

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1. *Isotropic to nematic transition*

   (a) Why is it that $\langle \hat{v}^{(n)} \rangle$ is not a good choice for order parameter for a nematic to isotropic transition? ($\hat{v}^{(n)}$ is a unit vector that points along the long axis of the *rod* like molecules.)

   (b) Argue that $Q_{ij} = \langle v_i^{(n)} v_j^{(n)} - \frac{1}{3}\delta_{ij} \rangle$ is an appropriate choice.

   (c) What is the form of Q in the isotropic phase? Justify

   (d) What is the diagonal form of Q in the nematic phase? Justify.

   (e) What do the eigenvectors of Q tell us about the system?

   [2 + 4 + 2 + 4 + 2]

2. *Markov process*

   (a) Explain briefly what is a Markov process?

   (b) Consider a continuous process in which the a variable, x, switches between two values, x_1 and x_2, randomly at rates $w_{12} = a$ and $w_{21} = b$. Write down the master equation for the process.

   (c) If $x = x_1$ at $t = 0$, what is the probability that the system is in state x_2 at $t = 1$?

   (d) Show that in equilibrium, the correlation function, $\langle \delta x(0) \delta x(t) \rangle = \frac{ab(x_1 - x_2)^2}{(a+b)^2} e^{-(a+b)t}$.

   [2 + 2 + 6 + 8]

3. *Brownian motion of harmonic oscillator:* Consider the over-damped Langevin equation for a particle in a harmonic potential,

$$m\eta v = -kx + f(t),$$

   where the noise $f(t)$ satisfies the conditions:
   $\langle f(t) \rangle = 0$ and $\langle f(t) f(t') \rangle = \Gamma \delta(t - t')$.

   (a) Find $\langle x(t) \rangle$ with $x(0) = x_0$.

   (b) Find $\langle \Delta x^2 \rangle = \langle x(t)^2 \rangle - \langle x(t) \rangle^2$ with $x_0 = 0$.

(c) Find the behaviour of $\langle \Delta x^2 \rangle$ for $t \ll \frac{m\eta}{k}$. Explain the result.

(d) Find the behaviour of $\langle \Delta x^2 \rangle$ for $t \gg \frac{m\eta}{k}$. Explain the result.

(e) Derive the fluctuation-dissipation theorem using the above result.

(f) Write down the corresponding Fokker-Planck equation for the process.

(g) What is the long time solution to the Fokker-Planck equation?

[4 + 4 + 2 + 2 + 2 + 2 + 2]

4. *Critical phenomena*

(a) High temperature series expansion for susceptibility of a spin system is given to be ($k_B = 1$):

$$\chi = 1 + \frac{4}{T} + \frac{12}{T^2} + \frac{34}{T^3} + \frac{88}{T^4} + ...$$

Find T_c and the susceptibility exponent, γ.

(b) What are scaling laws?

(c) How does Widom scaling hypothesis account for the scaling laws?

[6 + 2 + 2]