Birla Institute of Technology & Science – Pilani, K K Birla Goa Campus

Second Semester 2022-2023

Special Topics Statistical Mechanics Mid-semester Exam

PHY F423 March 13, 2023

Total Marks: 30 Duration: 90 Minutes

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1. *Inadequacy of mean field theory (MFT)*

   (a) Argue why MFT is not consistent close to the critical point.

   (b) What is Ginzburg criterion?

   (c) Explain what is upper critical dimension in the context of phase transitions.

   [3 + 2 + 2]

2. 1*D Ising model with spin* 1: Consider the Hamiltonian

$$H = -J \sum_{i=1}^{N} S_i S_{i+1} - h \sum_i S_i$$

   where S_i takes the values ± 1 and 0. Assume periodic boundary conditions with $S_{N+1} = S_1$.

   (a) Find the transfer matrix for the problem.

   (b) Explain how you would go about finding the internal energy of the system, starting with the transfer matrix. **[5 + 3]**

3. Consider the Landau free energy

$$F(m,T) = a(T) + \frac{b}{2}m^2 + \frac{c}{4}(T - T^*)m^4 + \frac{d}{6}m^6$$

   where $d > 0$ and b and $c(T - T^*)$ are larger than zero for high enough temperatures.

   (a) If it is given that $c(T - T^*)$ changes sign before b does as one comes down in temperature, argue that the system undergoes a first order transition.

   (b) Find the transition temperature.

   (c) Find the latent heat of transition. **[3 + 4 + 3]**

4. Consider the following variational free energy:

$$F_\rho = \frac{1}{2}\int\int d^3r d^3r' \rho(\mathbf{r})\rho(\mathbf{r}')U(\mathbf{r}-\mathbf{r}') + \int d^3r \rho(\mathbf{r})U_{\text{ext}}(\mathbf{r}) + k_B T \int d^3r \rho(\mathbf{r}) \text{Log}\left(\rho(\mathbf{r})\right)$$

   With the constraint $\int d^3r \rho(\mathbf{r}) = N$ in place, minimize the above trial free energy with respect to ρ to find an expression for density distribution. **[5]**